Dynamic Spreading Code Selection Method for PAPR Reduction in OFDM-CDMA Systems With 4-QAM Modulation

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***Abstract*—We present a new dynamic spreading code selection technique to obtain low peak-to-average-power ratio (PAPR) of an orthogonal frequency-division multiplexing code-division multiple-access (OFDM-CDMA) system with 4-QAM modulation for the down-link in mobile communication. In this method, initially, each user is assigned a low and a high PAPR spreading code, which allows selecting one when the system is operating. The spreading code of each user is dynamically selected so that total PAPR level of the whole user group present in the system is minimized. With the proposed technique, the PAPR level of a system with 10 users and 64 sub-carriers using 64 chip Walsh–Hadamard (WH) codes, can be limited to 15 dB while the worst case theoretical maximum could go up to 28.1 dB. Although 64 chip WH codes are employed to evaluate the performance it can be generalized to all other sets of spreading codes.**

***Index Terms*—CDMA, OFDM, PAPR, QAM.**

I. INTRODUCTION

An OFDM-CDMA system is attractive because of its efficient usage of bandwidth due to overlapping spectra and immunity to frequency selective fading [1]. However, OFDM-CDMA systems have the disadvantage of high PAPR. There are several methods such as peak amplitude limiting and coding [2], [3] proposed to reduce PAPR in OFDM, they are not very successful either due to bandwidth expansion or due to high complexity. Therefore we propose a dynamic spreading code allocation technique for downlink to reduce PAPR of an OFDM-CDMA significantly with 4-QAM modulation. A related technique of dynamically selecting sequences was presented in [4], which costs extra bandwidth. However, in this paper, we present how to implement the proposed idea in detail with PAPR bounds compared to an ordinary system. We propose to assign two spreading codes with high and low PAPR for each user and use two chip-matched correlators to identify which spreading code has been used at the transmitter.

In this method the spreading code set is divided into low PAPR codes which give PAPR below a specified level for a single user OFDM-CDMA system and high PAPR codes. After that each user is assigned a predetermined low and a high PAPR code. When the system is operating, according to the active user group present, for each user one spreading code is selected so that chosen code combination of the whole group leads to a minimum PAPR level of that group. As an example, if there are M active users in the group there are 2^{M} spreading code combinations to choose from. Therefore we can find at least one code combination with a low PAPR value. The spreading code combination with a low PAPR level for each user group needs to be found only once and stored in a database initially. As users enter and leave the system the relevant spreading code for each user can be searched and allocated dynamically using the database. At the receiver the demodulator can find which code was used in the transmitter using two chip-matched correlators corresponding to two spreading codes initially assigned for each user. Thus the demodulator can identify the information signal without transmitting any side information. The concept introduced can be applied with any set of spreading codes.

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II. SYSTEM MODEL

The OFDM-CDMA transmitter [5] used to describe the new PAPR reduction scheme for the downlink is described in this section. The input data symbols, $a_m[k]$, $b_m[k]$ are assumed to be binary antipodal and have a symbol duration of time T_s. Hence the symbol alphabet is 4-QAM. Here k denotes the kth symbol interval and m denotes the mth user. The symbols $a_m[k]$ and $b_m[k]$ are spread using a WH code of N chips of duration T_s. It is assumed that all users are synchronized. The discrete output signal samples of N point IDFT, with M active users present in the system, in kth OFDM symbol interval can be written as (spreading length = length of IDFT)

$$D[q,\, k] = \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} \{a_m[k]c_m[i] + jb_m[k]c_m[i]\}e^{j2\pi(iq/N)} \tag{1}$$

where $c_m[0], \ldots, c_m[N-1]$ represent the chips of the mth user's spreading code, $q = 0, 1, \ldots, N-1$, f_i is the frequency of the ith sub-carrier of an OFDM symbol and $f_i = f_0 + i/T_s$. Expression (1) is used to derive equations for PAPR of an OFDM- CDMA symbol in the next section.

A. PAPR of an OFDM-CDMA Symbol

In this section we will derive PAPR expressions in decibels for the transmitted OFDM-CDMA symbols of the proposed system. First we consider the definition of PAPR of an OFDM symbol given in [3]

$$\mathrm{PAPR} = 10\log_{10}\left\{\frac{0.5P_{\mathrm{peak}}}{P_{\mathrm{average}}}\right\}. \tag{2}$$

Since high PAPR values occur due to very high values of the P_{peak} given in (2) we will concentrate on P_{peak} of the OFDM-CDMA symbol in (1). If there are M number of active users in the system P_{peak} value of the transmitted symbol can be expressed as [3]

$$P_{\text{peak}} = \text{Max.}\left\{\left\{\text{Re}\left[\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\{a_m c_m[n] + j b_m c_m[n]\} e^{j2\pi f_n t}\right]\right\}^2\right\}, \quad (0 \le t < T_s),\ c_m[n] \in \{+1, -1\}, a_m \in \{+1, -1\},\ b_m \in \{+1, -1\} \tag{3}$$

where $c_m[n]$ is the nth chip of the mth user's spreading code and $a_m + jb_m$ is the 4-QAM symbol of the mth user before spreading. In (3) maximization should be evaluated over time interval $(0 < t < T_s)$ for all 4^M possible (a_m, b_m) data combinations of the 4-QAM modulator. In the single user case, expression (3) can be reduced into the following form:

$$P_{\text{peak}} = \text{Max.}\left\{\sum_{n=0}^{N-1} c_0[n] a_0 \cos(2p f_n t) - \sum_{n=0}^{N-1} c_0[n] b_0 \sin(2p f_n t)\right\}^2. \tag{4}$$

We will use expression (3) and (4) with all possible input data combinations of 4-QAM modulator to find PAPR and to describe the new dynamic spreading code allocation algorithm in the Section III.

III. The Dynamic Spreading Code Selection Method

First, consider a single user OFDM-CDMA transmitter to describe how to classify spreading code set into low and high PAPR codes. Since information signals $a_m[k]$ and $b_m[k]$ are binary values, only four input symbol combinations $(-1,-1), (-1,+1), (+1,-1), (+1,+1)$ will apply to any user's spreading code and to sub-carriers of an OFDM symbol. Therefore, it is possible to find the maximum PAPR value of these combinations for a particular spreading code for a single user OFDM-CDMA symbol using (4). If the maximum PAPR is larger than a predetermined level that spreading code is named as a high PAPR code. Therefore, "the length N WH code set" can be divided into low (PAPR below specified level) and high PAPR codes.

Next, a low PAPR (level below specified value L) code is assigned to each user. In the proposed system the second code of the user is assigned from the remaining high PAPR code set. In the third step, all users in the system are divided into pre-determined groups so that each group contains G (10) users. Since at any instant G or less than G users of a group can be present in the system, the fourth step is to find the spreading code combination with the lowest PAPR level correspond to that group. This is done considering all user combinations (1, 2, ..., G) within each group and store that information in a database. As an example if there are G users in a group there are 2^G spreading code combinations. Therefore using (3) we can find at least one code combination so that the combined PAPR level of that G users is very low. The low PAPR spreading code combination search method shows how to assign codes for all possible number of user combinations of the predetermined groups. The dynamic spreading code selection method shows how to pick the code combination dynamically for the users present in the system when it is operating as users leave and enter the system.

A. Low PAPR Spreading Code Combination Search

1) Consider M users. Assign each user a low PAPR (Level below L dB) and a high PAPR spreading code.
2) Divide M users into predetermined groups of $G(=10)$ users randomly.
3) Take each group of G users and follow steps a) – d) to build a database of the low PAPR code combinations for different number of users and user combinations [see b) below] in a group.
   a) Take each group of G users and find the code combination (out of 2^{10}) which gives PAPR level below the specified level [level between L and L_G as in (6)]. For each code combination only 2^{10} input data combinations need to be checked due to symmetrical properties of (3).
   b) Take all combinations of nine users $({}^{10}C_9)$. For each set of users find one-code combination (out of 2^9) which gives the PAPR level below the specified level (level between L and L_G). Each code combination has 2^9 input combinations to be checked in (3).
   c) Repeat step b) for 8, 7, ... up to 1 user(s) and find the code combination for each possible set of users which gives a PAPR level below specified level.
   d) Store each low PAPR code combination for each set of users in a database.

B. Dynamic Spreading Code Selection

1) Assume there are $m(0 < m \le M)$ active users in the system at the beginning. Divide m users into pre-determined group of G users.
2) Then select the low PAPR spreading code combination from the database corresponding to each user group and the number of users in that group.
3) Dynamically monitor whether users are leaving or entering the group. If so, go to step-2) and search for the new code combination relevant to the updated user set.

IV. PAPR Bound

According to (2) and (3), if the PAPR of each user is limited to a level below L dB, PAPR bound without the technique proposed above for a G user OFDM-CDMA system can be shown to be

$$L_G \le L + 10\log_{10} G. \tag{5}$$

In an ordinary system the maximum PAPR level of the G user OFDM-CDMA system could be as high as $10\log_{10}(GN)$ (N = sub-carriers). In the proposed method, if the PAPR of each user is limited to a level below L dB, a spreading code combination

TABLE I

Number of Users	PAPR in dB (Proposed System)	PAPR in dB (Worst case Ordinary System)
1	12(=L)	$10\log_{10}N$ (=18.1)
4	13	$10\log_{10}(4N)$ (=24.1)
G(=10)	15	$10\log_{10}(NG)$ (=28.1)
M[>G(=10)]	Less Than $L+10\log_{10}(M)$	$10\log_{10}(NM)$

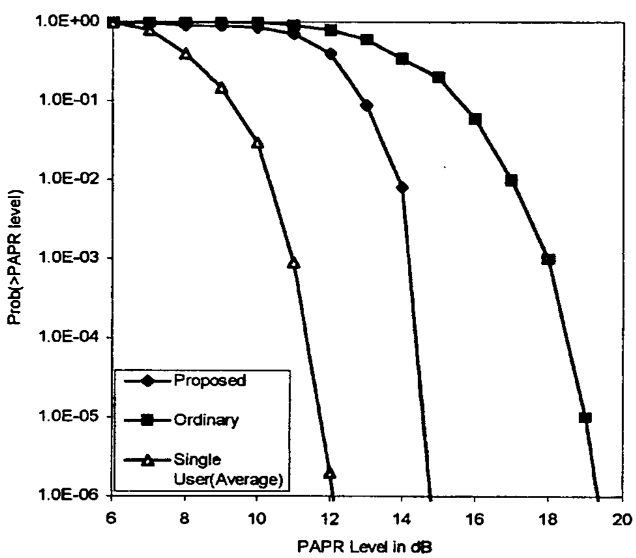

Fig. 1. Probability of signal samples exceeding a given PAPR level for proposed and ordinary systems of 10 user OFDM-CDMA symbols.

could be found so that in general PAPR bound lies in between L and L_G. For a known set of G spreading codes exact PAPR level can be evaluated although it varies for different spreading codes.

V. Performance Evaluation

In the example system all users are grouped into pre-determined $G(=10)$ user groups. Each user is assigned 2 WH spreading codes of length 64, of PAPR level below and above 12 dB. Table I given below shows a comparison of actual PAPR levels obtained for a 64-chip WH code set using a computer search. The OFDM symbol was over-sampled 5 times the Nyquist rate in order to miss any P_{peak} and all possible information bit combinations to the transmitter were considered.

The row 5 of Table I. Shows the maximum PAPR level that can occur for both systems although the probability of occurrence close to that level is very small. For the proposed system the expression has been derived using (5). If there are M users, in the proposed system and individual user's PAPR level is limited to L, according to (5) we can always find code combinations which gives PAPR level less than $(L + 10\log_{10}(M))$ after dividing users into G user groups.

Fig. 1 shows the probability of signals exceeding a given PAPR level for both the proposed and ordinary systems evaluated using the approach in [4]. For the ordinary system with the simulation results have been obtained by randomly choosing 10 WH codes of length 64 $({}^{64}C_{10})$, as spreading codes with average PAPR level of 12 dB. These codes were assigned to tenusers and added to get the OFDM-CDMA transmit signal. The results show that the PAPR could go up to 19 dB with 10^{-6} probability although the worst case PAPR level could go up to 28.1 dB. After adding 10 users even high PAPR level is obtained since WH codes have less randomness properties compared to other spreading code sets. The worst case PAPR occurs when all 10 WH codes were selected from the highest PAPR codes of the set.

VI. Conclusions

The proposed method can be seen as a technique to avoid high PAPR in OFDM-CDMA with 4-QAM signaling. The actual PAPR reduction depends on the system capacity needed. The maximum number of users is equal to half of the spreading code set. This method can be generalized to any other code set such as Gold codes by appropriately choosing N (31, 63..., for Gold codes) and the total number of users in the system. The overall PAPR level obtained is not the absolute minimum since the size of G can be less than the total number of users in the system. However, as seen in Table I, PAPR levels in the two systems increases significantly with the number of users.

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References

[1] S. Kaiser, "OFDM/CDMA versus DS/CDMA: Performance evaluation for fading channels," in *VTC'95*, pp. 1722–1726.

[2] D. Wulich, "Reduction of peak to mean power ratio of multi-carrier modulation using cyclic coding," *Electron. Lett.*, vol. 32, no. 5, pp. 432–433, Feb. 29, 1996.

[3] D. Wulich and L. Goldfeld, "Reduction of peak factor in orthogonal multi-carrier modulation by amplitude limiting and coding," *IEEE Trans. Commun.*, vol. 47, pp. 18–21, Jan. 1999.

[4] H. Ochiai and H. Imai, "OFDM-CDMA with peak power reduction based on the spreading sequences," in *ICC'98*, vol. 3, pp. 1299–1303.

[5] T. Ginige, N. Rajatheva, and K. M. Ahmed, "Performance of an OFDM-CDMA system with low peak to average power ratio," in *WPMC'00*, vol. 1, Thailand, Nov. 12–16, pp. 39–44.